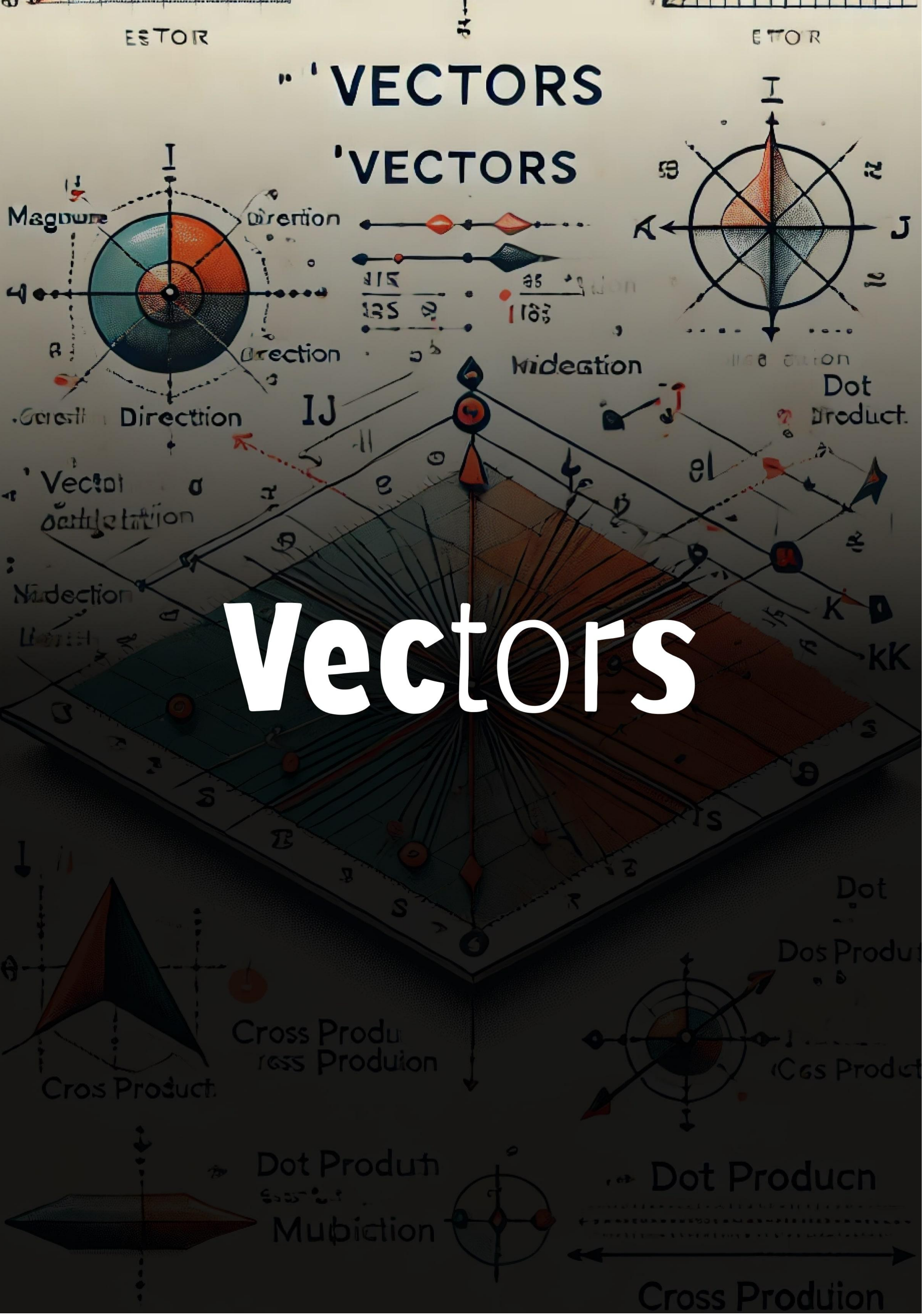


Vectors

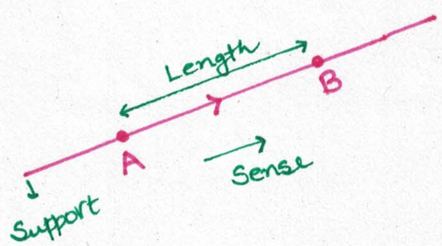
VECTORS

DEFINATION Vector is a quantity which contains direction as well as magnitude and satisfying the addition law of vector.

DIRECTED LINE SEGMENT The portion of the line whose 2 end points are specified as initial and terminal points is known as DLS.

A DLS with A as initial pt. and B as terminal pt. is known as $\overrightarrow{AB}$

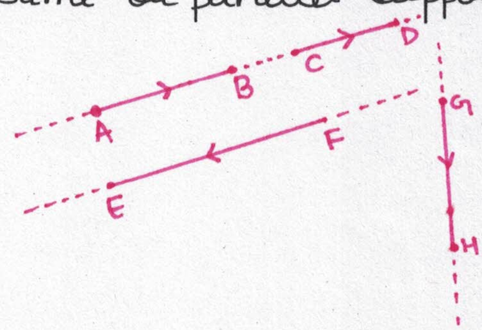
A DLS consists of 3 parts:-



- **Length** — Magnitude of vector
- **Sense** — Shows the direction of vector from initial to terminal
- **Support** — The line of unlimited length, whose portion is DLS

The question of comparison of sense of 2 vectors arises when they have the same or parallel support.

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ are antiparallel to $\overrightarrow{EF}$
They have same or parallel support
But we cannot compare this sense with $\overrightarrow{GH}$ as it does not have parallel support

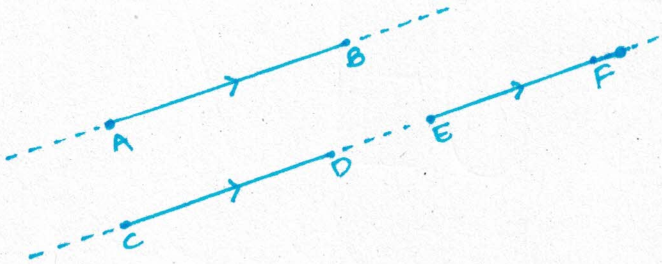


BASIC FUNDAMENTALS

1 Equality of 2 vectors

Two vectors are said to be equal if-

(i) Their length are same
(ii) Their sense are same
(iii) Support same or parallel



$\vec{AB} = \vec{CD} = \vec{EF}$

$|\vec{AB}| = |\vec{CD}| = |\vec{EF}|$

(2) Null Vector or Zero Vector ($\vec{0}$)

A vector having magnitude zero and direction not specified is called null vector.

Geometrically, a null vector is a pt. and mathematically, it shows similarity with the digit '0'.

A null vector is collinear with every vector in space and all the lines are support to null vector.

(3) Unit Vector

A vector of unit magnitude and directed along a given vector $\vec{a}$ is called unit vector in the direction of $\vec{a}$ and represented as-

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

The concept of unit vector is to impart the direction to a physical quantity, i.e. unit vector is used to represent the direction of physical quantity

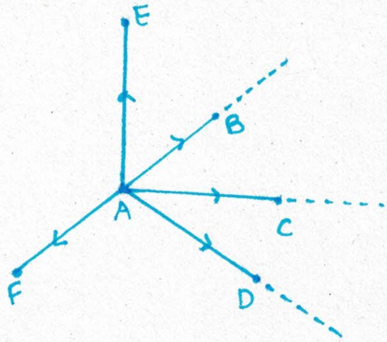
(4) Free and Localised vector

A vector is said to be free vector which when transformed into space, from one point to another without any change in physical quantity (without change in length and sense) is known as free vector.

- The physical effect will not affect if we shift the line of action of a free vector parallel. eg. - Velocity
- The physical effect changes if the line of action is changed of a given vector and that vector is known as localised vector eg. - Force

(5) Coinitial Vectors

All the vectors having same initial points are called coinitial vectors. Their lengths can be differed, sense can be differ and support can differ but initial point is same.

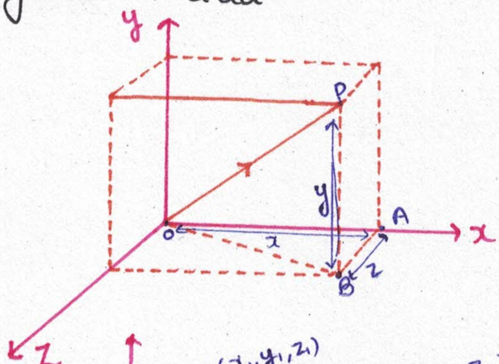


$\vec{AB}, \vec{AC}, \vec{AD}, \vec{AE}, \vec{AF}$ are coinitial vectors.

If the terminal pt. of 2 coinitial vectors are same, then they are known as equal vectors.

6 Position Vectors

Let O be the origin, then position vector of a pt. P will be given as $\vec{OP}$ which specify the position of an object in 3-D space, also known as Orthogonal Triad.

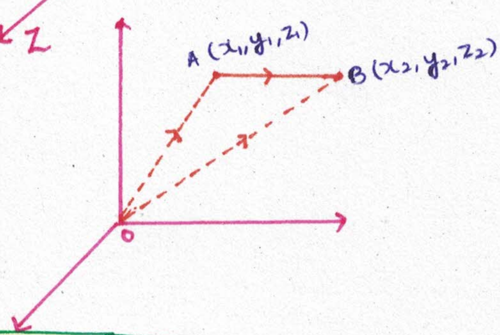


$\vec{OA} + \vec{AB} = \vec{OB}$

$x\hat{i} + z\hat{k} = \vec{OB}$

$\vec{OB} + \vec{BP} = \vec{OP}$

$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$



$\vec{OA} + \vec{AB} = \vec{OB}$

$\vec{AB} = \vec{OB} - \vec{OA}$

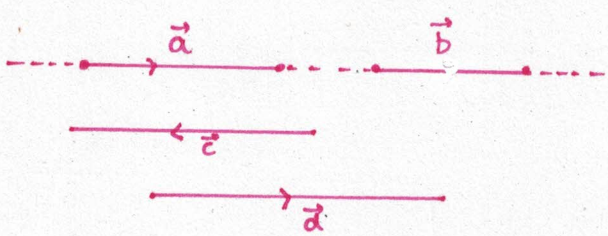
$= (P.V.)_{\vec{B}} - (P.V.)_{\vec{A}}$

$|\vec{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

7 Parallel Vectors

Two vectors are said to be parallel or collinear vector if their DLS are either parallel or same, irrespective of their sense.

If the sense are same, then they are called like vectors and if sense are opposite, they are called unlike vectors.



$\vec{a}, \vec{b}, \vec{c}, \vec{d} \rightarrow$ parallel or collinear

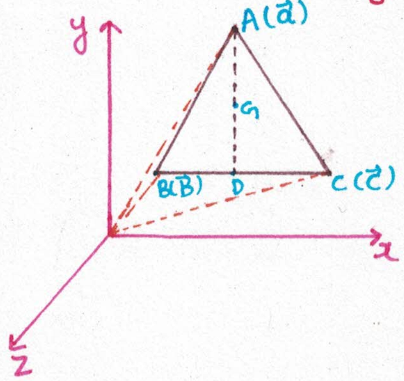
$(\vec{a}, \vec{d})\ (\vec{b}, \vec{c}) \rightarrow$ Like

$(\vec{a}, \vec{c})\ (\vec{a}, \vec{b})\ (\vec{c}, \vec{d}) \rightarrow$ Unlike

Question : If a vertex of the ΔABC are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

Find the centroid of ΔABC.

Sol:



$$\vec{G} = \frac{\vec{a}+\vec{B}+\vec{c}}{3}$$

Proof:

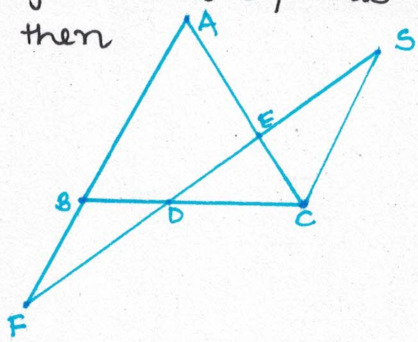
$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$\vec{G} = \frac{(x_1+y_1+z_1)\hat{i} + (x_2+y_2+z_2)\hat{j} + (x_3+y_3+z_3)\hat{k}}{3}$$

$$\vec{G} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

Theorem 1 : Milanius Theorem

If we take 3 points on the sides of the ΔABC which are collinear, then



$$\frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{FB} = 1$$

$\Delta BDF \sim \Delta CDS$

$$\frac{BD}{DC} = \frac{FB}{CS}$$

$\Delta CSE \sim \Delta AFE$

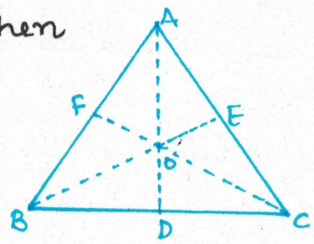
$$\frac{CE}{AE} = \frac{CS}{AF}$$

$$\frac{BD}{DC} \times \frac{CE}{AE} = \frac{FB}{CS} \times \frac{CS}{AF}$$

$$\Rightarrow \boxed{\frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{FB} = 1}$$

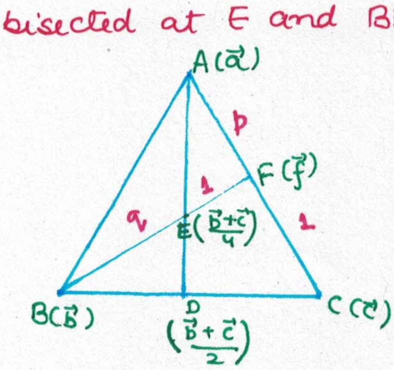
Theorem 2: Lena's Theorem

If O is a pt. inside the ΔABC and we connected the vertex of Δ to 'O' which meets the opposite side at D, E and F respectively, then



$$\boxed{\frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{FB} = 1}$$

Question: The median AD of the ΔABC is bisected at E and BE meets AC at F. Then find ratio of $\frac{AF}{AC}$.



Sol:

$$\frac{\vec{b}+\vec{c}}{4} = \frac{q\vec{f}+\vec{b}}{q+1}$$

$$(-1+q)\vec{f} = \left(\frac{\vec{b}+\vec{c}}{4}\right)\vec{q} - \vec{b}$$

$$\vec{f} = \frac{\left(\frac{\vec{b}+\vec{c}}{4}\right)\vec{q} - \vec{b}}{q-1}$$

$$\Rightarrow \vec{f} = \frac{p\vec{c}}{p+1} = \frac{\left(\frac{\vec{b}+\vec{c}}{4}\right)\vec{q} - b}{q-1}$$

$$\Rightarrow \left(\frac{p}{p+1}\right)\vec{c} = \frac{\left(\frac{q}{4}\right)\vec{c}}{q-1} - \frac{\left(\frac{3\vec{b}}{4}\right)q}{q-1}$$

$$\Rightarrow \frac{p}{p+1} = \frac{q}{4(q-1)}$$

$$\Rightarrow 4(pq - p) = pq + q$$

$$\Rightarrow 3pq = q + 4p$$

$$\Rightarrow p(3q-4) = q$$

$$\Rightarrow p = \frac{q}{3q-4}$$

$q = 1$
$p = 1/2$
$AF : AC = 1:3$

ALGEBRA OF VECTORS

Multiplication of a vector by a scalar:

If $\vec{a}$ is a vector and λ is a scalar, then $\lambda\vec{a}$ is a parallel vector to $\vec{a}$ whose magnitude is λ times of $\vec{a}$

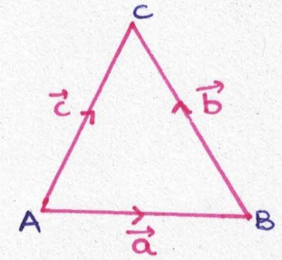
* $\lambda\vec{a} = \vec{a}(\lambda)$
* $\lambda(\mu\vec{a}) = \mu(\lambda\vec{a}) = \mu\lambda\vec{a}$
* $\lambda(\vec{a}+\vec{b}) = \lambda\vec{a} + \lambda\vec{b}$
* $\vec{a}(\mu+\lambda) = \lambda\vec{a} + \mu\vec{a}$

Addition of 2 vectors:

1. Triangle Law

$\vec{AB} + \vec{BC} = \vec{AC}$

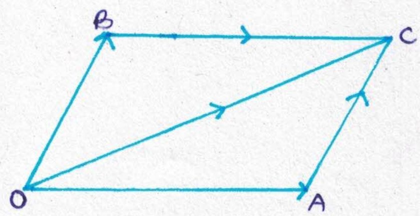
{Note : In ΔABC, $\vec{AB} + \vec{BC} + \vec{CA} = 0$}



2. Parallelogram Law

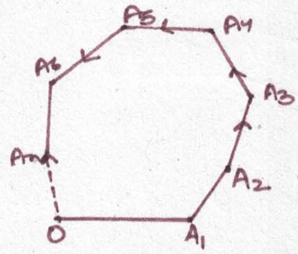
$\vec{OA} + \vec{AC} = \vec{OC}$

or $\vec{OA} + \vec{OB} = \vec{OC}$



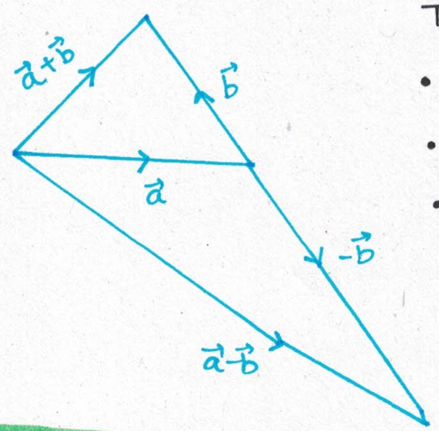
3. Polygon Law

$$\vec{OA_1} + \vec{A_1A_2} + \vec{A_2A_3} + \dots \vec{A_{n-1}A_n} = \vec{OA_n}$$



PROPERTIES

1. $\vec{a} + \vec{0} = \vec{a}$
2. Additive Inverse : $\vec{a} + \vec{b} = 0 \Rightarrow \vec{a} = -\vec{b}$
   Here, $\vec{b}$ is called additive inverse of $\vec{a}$
3. Commutative Property : $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
4. Associative Property : $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
5. Substraction



Triangle Inequalities

- $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$
- $|\vec{a} - \vec{b}| \geq |\vec{a}| - |\vec{b}|$
- $|\vec{a} + \vec{b}| \geq ||\vec{a}| - |\vec{b}||$
- $|\vec{a} - \vec{b}| \leq ||\vec{a}| + |\vec{b}||$

Ques: If the sum of 2 unit vectors is a unit vector, then find the angle b/w $\vec{a}$ and $\vec{b}$ and magnitude of their difference.

Sol:

$1 = \sqrt{1+1+2\cos\theta}$

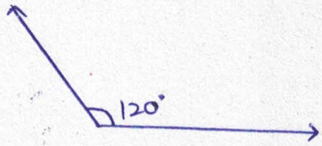
$1 = 2 + 2\cos\theta$

$-\frac{1}{2} = \cos\theta$

$\theta = 120°$

$|a-b| = \sqrt{1+1+2(1/2)}$

$|a-b| = \sqrt{3}$



Linearly Dependent and Independant System

Let $\vec{a_1}, \vec{a_2}, \vec{a_3} \dots \vec{a_n}$ are 'n' different vectors given in space, then we can find a vector 'V' which is a linear combination of all the given vectors i.e. -

$$\vec{V} = \lambda_1\vec{a_1} + \lambda_2\vec{a_2} + \lambda_3\vec{a_3} + \dots + \lambda_n\vec{a_n}$$

- If $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \lambda_3\vec{a}_3 + \dots\dots + \lambda_n\vec{a}_n = \vec{0}$

  implies $\lambda_1 = \lambda_2 = \lambda_3 = \dots = \lambda_n = 0$

  then, this system of vectors is known as linearly independent system.

  → Two non-parallel coplanar vectors

- If $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \dots\dots + \lambda_n\vec{a}_n = \vec{0}$

  Implies at least two of $\lambda_1, \lambda_2, \dots \lambda_n \neq 0$.

  Then, this system of vectors is known as linearly dependent system.

  → Two parallel vectors

In order to test whether the given system of vectors is linearly dependent or independent, we use following steps:

Step- I: Let $\vec{a}$ and $\vec{b}$ are two given vectors, then first write,

$$\vec{r} = x\vec{a} + y\vec{b}$$

$x, y \in$ scalar quantities and $\vec{r}$ is a LC of $\vec{a}$ and $\vec{b}$

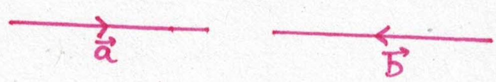
Step-II: Now, check to make $\vec{r} = \vec{0}$, if we require to make both x and y, 0 or not.

If yes, then system is linearly independent or else dependent.

Testing of Two vectors

Two vectors in a space are always coplaner

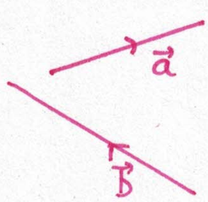
Case-I Two collinear vector



$\vec{r} = x\vec{a} + y\vec{b} = \vec{0}$ $\quad x = 1 \quad y = \frac{|a|}{|b|}$

Linearly Dependent

Case-II Two non-parallel vector



$\vec{r} = x\vec{a} + y\vec{b} = 0$

Possible only when $x = y = 0$

Linearly Independent system

These 2 vectors will always give some resultant vector

Testing of 3 vectors

Case- I 3 non-coplanar vectors

$$x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$$

$x = 0 = y = z$

Linearly Independent

Case- II 3 coplanar vectors

Linearly Dependent

NOTE 4 vectors always form linearly dependent system

Ques: $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ $\vec{b} = 2\hat{i}+6\hat{j}-\hat{k}$ $\vec{c} = 9\hat{i}-\hat{j}+3\hat{k}$

Prove that they form a linearly independent system.

Sol:
$$\vec{r} = x(\hat{i}+\hat{j}+\hat{k}) + y(2\hat{i}+6\hat{j}-\hat{k}) + z(9\hat{i}-\hat{j}+3\hat{k})$$
$$\hat{i}(x+2y+9z) + \hat{j}(x+6y-7) + \hat{k}(x-y+3z) = \vec{0}$$

$$\begin{array}{l} x+2y+9z = 0 \\ x+6y-7 = 0 \\ \hline -4y+10z = 0 \end{array} \qquad y = \frac{10z}{4}$$

$$\begin{array}{l} x+6y-z = 0 \\ x-y+3z = 0 \\ \hline 7y-4z = 0 \end{array} \qquad y = \frac{4z}{7}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 6 & -1 \end{vmatrix} = \hat{i}(-7) - \hat{j}(-1-2) + \hat{k}(6-2)$$
$$= -7\hat{i} + 3\hat{j} + 4\hat{k}$$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = (-7\hat{i}+3\hat{j}+4\hat{k})\cdot(9\hat{i}-\hat{j}+3\hat{k}) = -63-3+12 \neq 0$$

If box product is zero, 3 vectors are coplanar

$[\vec{a}\,\vec{b}\,\vec{c}] \neq 0 \Rightarrow$ These 3 vectors were non coplanar and non-coplanar vectors are linearly independent

$\hat{a} = \hat{i}+\hat{j}$ $\hat{b} = \hat{j}+\hat{k}$ $\hat{c} = \hat{i}+\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(1) + \hat{k}(1)$$

$$= (\hat{i}+\hat{k})(\hat{i}-\hat{j}+\hat{k}) = 1+1 = 2$$

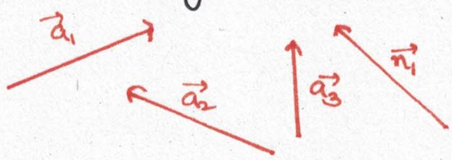
Non- coplanar, Linearly Independent

If determinant is zero $\rightarrow$ Coplanar vectors

COPLANER VECTOR A set of vectors is said to be coplanar vector if their supports are parallel to same plane.

i.e. $\vec{n}_1 \cdot \vec{a}_1 = \vec{n}_2 \cdot \vec{a}_2 = \ldots\ldots = \vec{n}_n \vec{a}_n = \vec{0}$

Then, $\vec{a}_1, \vec{a}_2, \ldots\ldots \vec{a}_n$ are said to be coplanar where, $\vec{n}_1$ is the normal vector to the plane containing $\vec{a}_1, \vec{a}_2, \ldots\ldots \vec{a}_n$



NOTE

Three points with parallel vector $\vec{a}, \vec{b}, \vec{c}$ are collinear if there exist scalar quantities x, y and z such that-

(i) $x\vec{a} + y\vec{b} + z\vec{c} = 0$

(ii) $x + y + z = 0$

(iii) not all x, y, z are zero

NOTE

Four points with parallel vector $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar if there exist four scalars x, y, z and t

(i) $x\vec{a} + y\vec{b} + z\vec{c} + t\vec{d} = 0$

(ii) $x + y + z + t = 0$

(iii) Sum of any two scalars is not zero

Ques: $\vec{a}, \vec{b}, \vec{c}$ are non-zero vectors no two of which are collinear if $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then find $\vec{a} + 2\vec{b} + 3\vec{c}$.

Sol: $\vec{a} + 2\vec{b} = \lambda\vec{c}$ $\qquad\qquad 2(\vec{b} + 3\vec{c} = \mu\vec{a})$

$\vec{a} - 6\vec{c} = \lambda\vec{c} - 2\mu\vec{a}$

$\vec{a}(1 + 2\mu) = \vec{c}(\lambda + 6)$

$\mu = -\frac{1}{2} \qquad \lambda = -6$

$\vec{a} + 2\vec{b} = -6\vec{c} \qquad\qquad 2\vec{b} + 6\vec{c} = -\vec{a}$

$\vec{a} + 2\vec{b} + 3\vec{c} = -6\vec{c} + 3\vec{c}$

$\vec{a} + 2\vec{b} + 3\vec{c} = |-3\vec{c}|$

$= |3\vec{c}|$

POINTS TO REMEMBER

① Linearly independent implies no vector can be expressed as a linear combination of other.

② Linearly dependent implies that at least one vector can be expressed as a LC of remaining vector.

$$x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$$

$$\vec{a} = -\frac{y}{x}\vec{b} - \frac{z}{x}\vec{c} \qquad x, y, z \neq 0$$

$$\vec{a} = \lambda_1\vec{b} + \lambda_2\vec{c}$$

Ques: $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors in such a way that every pair is non-collinear and $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ are collinear with $\vec{c}$ and $\vec{a}$ respectively, find $\vec{a}+\vec{b}+\vec{c}$.

Sol: $\vec{a} + \vec{b} = \lambda\vec{c} \qquad \vec{b} + \vec{c} = \mu\vec{a}$

$$\vec{a} - \vec{c} = \lambda\vec{c} - \mu\vec{a}$$

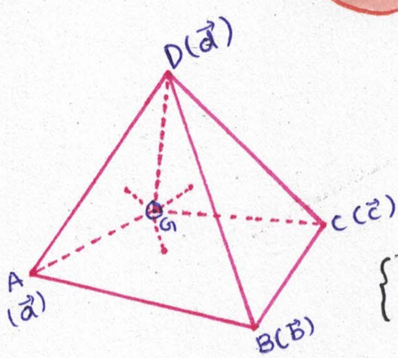
$$\vec{a}(1+\mu) = \vec{c}(1-\lambda)$$

$$\mu = -1 \qquad \lambda = -1$$

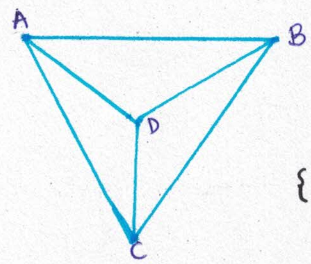
$$\vec{a} + \vec{b} = -\vec{c}$$

$$\vec{a} + \vec{b} + \vec{c} = 0$$

TETRAHEDRON



{Tetrahedron Side View



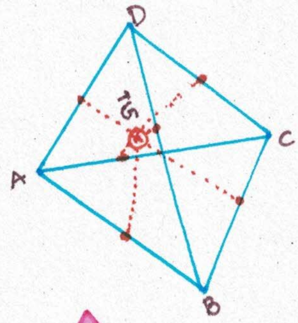
{Top View

$$\boxed{\vec{G} = \frac{\vec{a} + \vec{b} + \vec{c} + \vec{d}}{4}}$$

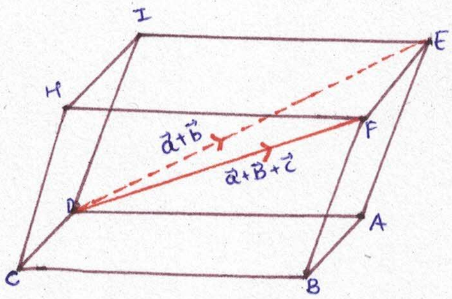
- It is a pyramid of triangular base
- It has 4 triangular faces
- It has pair of opposite edge :-
  (AB, OC) (BC, OA) (AC, OB)
- The line joining the vertex to the opposite triangular face centroid

are concurrent and POC is called centroid of tetrahedron

- The line joining the mid pt. of opposite edge are also concurrent at $\vec{G}$



PARALLELEPIPED



A prism whose base is a parallelogram and all faces are parallelogram.

$$\vec{G} = \frac{\vec{a}+\vec{b}+\vec{c}}{2}$$

- Four diagonals of a parallelopiped are concurrent, at the centre of the parallelepiped and is bisected by the centre
- The line joining the mid pt. of each pair of opposite edge are also concurrent at the centre of parallelepiped.

Ques: If the centre of the parallelepiped formed by $\vec{PA}, \vec{PB}$ and $\vec{PC}$, where $\vec{PA}$ equals $\hat{i}+2\hat{j}+2\hat{k}$, $\vec{PB} = 4\hat{i}-3\hat{j}+\hat{k}$, $\vec{PC} = 3\hat{i}+5\hat{j}-\hat{k}$ is given by $7\hat{i}+6\hat{j}+2\hat{k}$, then find the P.V. of P?

Sol:
$$\frac{\vec{PA}+\vec{PB}+\vec{PC}}{2} = \frac{8\hat{i}+4\hat{j}+2\hat{k}}{2}$$

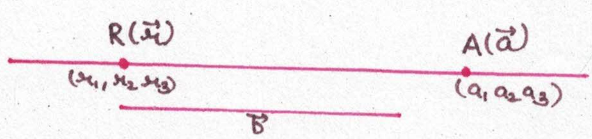
$$\vec{PR} = 4\hat{i}+2\hat{j}+\hat{k}$$

$$\vec{OR} = 7\hat{i}+6\hat{j}+2\hat{k}$$

$$\vec{P} = 3\hat{i}+4\hat{j}+\hat{k}$$

Vector Equations Of a Line

Type-1 The equation of line passing through $\vec{a}$ and parallel to $\vec{b}$



$$\vec{AR} = \lambda\vec{b}$$

$$\vec{r}-\vec{a} = \lambda\vec{b}$$

$$\boxed{\vec{r} = \vec{a}+\lambda\vec{b}}$$

$$\langle x, y, z\rangle = \langle a_1 a_2 a_3\rangle + \lambda\langle b_1 b_2 b_3\rangle$$

$x = a_1+\lambda b_1$ $\quad$ $y = a_2+\lambda b_2$ $\quad$ $z = a_3+\lambda b_3$

POINTS TO REMEMBER

1. The vector eq. of a line $\vec{r} = \vec{a}+\lambda\vec{b}$ can also be represented as $\vec{r} = \langle a_1 a_2 a_3\rangle + \lambda\langle b_1 b_2 b_3\rangle$
2. The line $\vec{r} = \vec{a}+\lambda\vec{b}$ represents the line passing through $\vec{a}$ and parallel to $\vec{b}$
3. The parametric co-ordinate of any pt. P on the line $\vec{r} = \vec{a}+\lambda\vec{b}$ will be given as-

   $\langle a_1+\lambda b_1, a_2+\lambda b_2, a_3+\lambda b_3\rangle$
4. Two lines in a plane are either intersecting or parallel, and conversely 2 intersecting or parallel lines are always coplanar
5. If the lines are parallel and having a common point, then they are coincident
6. In 3D, a line can be parallel or intersecting or non-parallel non-intersecting (skew lines) or (non coplanar lines)

Ques: How to find the POI of 2 intersecting lines?

Step I: To find the parallel vector of POI of 2 lines, find the parametric co-ordinate of each line and equate their coefficient of $\hat{i}, \hat{j}, \hat{k}$

Step II: We'll get 3 equation in 2 variables

Step III: Solve any 2 equations to find the values of variables and Satisfy them in 3rd equation

Step IV: If the variables satisfy the 3rd eq. then the lines are inter-secting, if does not satisfy, then lines are skew. If it makes equation an identity, then the lines are coincident.

Ques: $\vec{r} = \langle 1, -1, -10\rangle + \lambda\langle 2, -3, 8\rangle$

$\vec{r} = \langle 4, -3, -1\rangle + \mu\langle 1, -4, 7\rangle$

$1+2\lambda = 4+\mu$ $\qquad\qquad$ $-1-3\lambda = -3-4\mu$

$2\lambda-\mu = 3 \quad — ①$ $\qquad\qquad$ $4\mu-3\lambda = -2 \quad — ②$

$$\begin{array}{r} 8\lambda-4\mu = 12 \\ 4\mu-3\lambda = -2 \\ \hline 5\lambda = 10 \end{array} \Rightarrow \lambda = 2, \quad \mu = 1 \quad \text{(Intersecting Lines)}$$

$-10+8\lambda = -10+16 = 6$

$-1+7\mu = -1+7 = 6$

$POI = \langle 5, -7, 6 \rangle$

Ques: $\vec{r} = \langle -3, 6, 0 \rangle + \lambda \langle -4, 3, 2 \rangle$

$\vec{r} = \langle -2, 0, 7 \rangle + \mu \langle -4, 1, 1 \rangle$

$-3-4\lambda = -2-4\mu$ $\qquad\qquad$ $6+3\lambda = -4\mu$

$$\begin{array}{r} 4\mu-4\lambda = 1 \\ -4\mu-3\lambda = 6 \\ \hline -7\lambda = 7 \end{array} \Rightarrow \lambda = -1$$

$3\lambda+4\mu = -6$

$4\mu+4 = 1$

$\mu = -3/4$ $\qquad$ (Skew Lines)

Ques: $\vec{r} = \lambda(3, -1, -1)$

$\vec{r} = \langle 2, 0, 0 \rangle + \mu \langle -6, 2, -2 \rangle$

$3\lambda = 2-6\mu$ $\qquad$ $-\lambda = 2\mu$

$-6\mu = 2-6\mu$ $\qquad$ $6\mu = 2-6\mu$ $\qquad$ $\mu = 1/6$ $\qquad$ (Skew lines)

Ques: $\vec{r} = \langle 0, 0, 2 \rangle + \lambda \langle 3, 2, 1 \rangle$

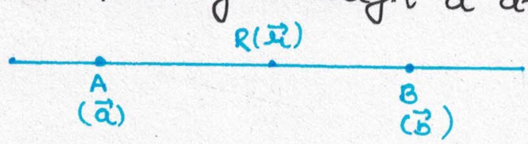
$\vec{r} = \langle 3, 2, 3 \rangle + \mu \langle 6, 4, 2 \rangle$

$3\lambda = 3+6\mu$ $\qquad$ $2\lambda = 2+4\mu$ $\qquad$ $2+\lambda = 3+2\mu$

$\lambda = 1+2\mu$ $\qquad$ $\lambda = 1+2\mu$ $\qquad$ $\lambda = 1+2\mu$

(Coincident Lines)

Type - II Equation of line passing through $\vec{a}$ and $\vec{b}$



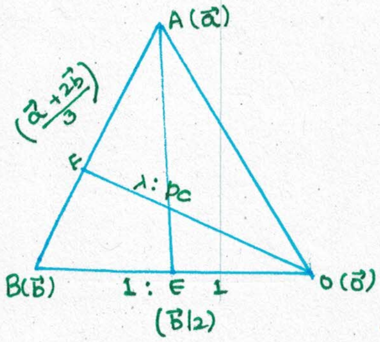
$\vec{AR} = \lambda \vec{AB}$

$\vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$

$$\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$$

Ques: In a Δ, AOB where O is origin, E is the mid pt. of OB. F divides BA in 1:2 ratio, then find OP/PF ? (P is POI of OF and AE)

Sol:



$$F = \frac{1(\vec{a}) + 2(\vec{b})}{3}$$

OF $\vec{r} = \vec{o} + \lambda\left(\frac{\vec{a}+2\vec{b}}{3}\right)$

AE $\vec{r} = \vec{a} + \mu\left(\frac{\vec{b}}{2} - \vec{a}\right)$

$$\lambda\left(\frac{\vec{a}+2\vec{b}}{3}\right) = \vec{a} + \mu\left(\frac{\vec{b}}{2} - \vec{a}\right)$$

$\frac{\lambda}{3} = 1-\mu$ $\qquad$ $\frac{2\lambda}{3} = \frac{\mu}{2}$

$\mu = 1 - \frac{\lambda}{3}$ $\qquad$ $\lambda = \frac{3}{5}$

$$\vec{p} = \vec{o} + \frac{3}{5}\left(\frac{\vec{a}+2\vec{b}}{3}\right)$$

$$\vec{p} = \frac{\lambda(0) + \frac{\vec{a}+2\vec{b}}{3}}{\lambda+1}$$

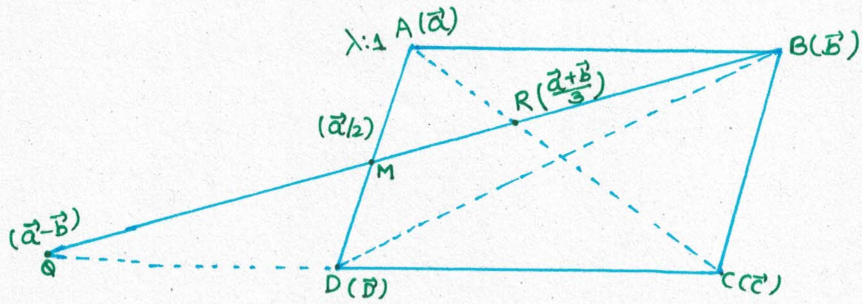
$\lambda + 1 = 5/3$

$\lambda = 2/3$

$$\boxed{\frac{OP}{PF} = 3:2}$$

Ques: Through the middle pt. M of side AD of a parallelogram ABCD, a straight line BM is drawn intersecting AC at R and CD produced at Q. Find $\frac{QR}{RB}$?

Sol:



$\vec{AC} = \vec{a} - \lambda(\vec{a} - \vec{c})$

$\vec{BM} = \vec{b} - \mu\left(\vec{b} - \frac{\vec{a}}{2}\right)$

$\vec{a} - \lambda(\vec{a} - (\vec{b} - \vec{a}))$ $\qquad$ $\vec{c} + \vec{a} = \vec{b}$ $\qquad$ $R = \vec{b} - \frac{2}{3}\vec{b} + \frac{\vec{a}}{3}$

$= \vec{b} - \mu\left(\vec{b} - \frac{\vec{a}}{2}\right)$ $\qquad$ $\lambda = 1-\mu$ $\qquad$ $R = \frac{\vec{b}}{3} + \frac{\vec{a}}{3}$

$-2\lambda + 1 = \mu/2$

$\frac{3\mu}{2} = 1$ $\qquad$ $\mu = \frac{2}{3}$

$\vec{DC} = \vec{o} + \lambda(\vec{c})$

$\lambda(\vec{b} - \vec{a}) = \vec{b} - \mu\left(\vec{b} - \frac{\vec{a}}{2}\right)$

$\lambda = 1 - \mu$

$-\lambda = \mu/2$

$-\frac{\mu}{2} + \mu = 1$

$\mu = 2$

$Q = \vec{b} - 2\vec{b} + \vec{a} = \vec{a} - \vec{b}$

$RB = \left(-\vec{b} - \left(\frac{\vec{a}+\vec{b}}{3}\right)\right) = \left(\frac{2b^{-1} - a^{-1}}{3}\right)$

$QR = \frac{\vec{a} + \vec{b} - 3\vec{a} + 3\vec{b}}{3}$

$R = \left(\frac{b\lambda + \vec{a} - \vec{b}}{\lambda + 1}\right) = \frac{\vec{a} + \vec{b}}{3}$

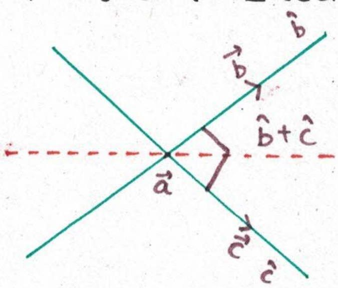
$3\ (\lambda - 1) + 3\vec{a} = \lambda\vec{a} + \lambda\vec{b} + \vec{a} + \vec{b}$

$3 = \lambda + 1$

$\lambda = 2$ $\boxed{2:1}$

Vector Equation of Angle Bisector

Let there are 2 vector equation of line $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{a} + \mu\vec{c}$



$\vec{r} = \vec{a} + s(\hat{b} + \hat{c})$

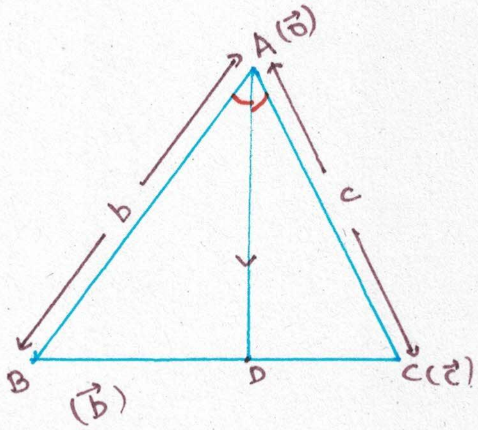
$\vec{r} = \vec{a} + p(\hat{b} - \hat{c})$

Ques: Use vector to prove that internal bisector of a Δ divides the opposite base in the ratio of their corresponding sides.

Sol: $\vec{AD} = \vec{0} + \lambda(\hat{b} + \hat{c})$

$\vec{D} = \vec{b} + \mu(\vec{b} - \vec{c})$

$\lambda(\hat{b} + \hat{c}) = \vec{b} + \mu(\vec{b} - \vec{c})$



$\lambda = \frac{1}{b} + \frac{\mu}{b}$ $\qquad \lambda = -\frac{\mu}{c}$

$\lambda = -\frac{1}{b} - \frac{\lambda c}{b} \Rightarrow \lambda b + \lambda c = -1$

$\lambda = \frac{-bc}{b+c}$

$D = \frac{-bc}{b+c}(\hat{b} + \hat{c})$

$= -\frac{bc}{b+c}\left(\frac{\vec{b}}{b} + \frac{\vec{c}}{c}\right)$

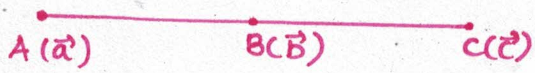
$= -\frac{b\vec{b} + c\vec{c}}{b+c}$

HENCE PROVED

Condition of Collinearity of 3 points $\{A(\vec{a}), B(\vec{b}), C(\vec{c})\}$

⊙ Let A, B, C are collinear, then it is possible if and only if there exist 3 scalars x, y, z (not all 0 at a time) such that $x\vec{a}+y\vec{b}+z\vec{c}=\vec{0}$ and $x+y+z=0$

⊙ Let A, B, C are collinear



$\vec{AB} = \lambda\,\vec{BC}$

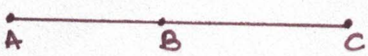
$\vec{b}-\vec{a} = \lambda(\vec{c}-\vec{b})$

$\vec{a} + (-\lambda-1)\vec{b} + \lambda\vec{c} = 0$

(I) $x=1,\ y=-\lambda-1,\ z=\lambda$

$x+y+z=0$

(II)



$\vec{b} = -\left(\dfrac{x\vec{a}+z\vec{c}}{y}\right)$

$\vec{b} = \left(\dfrac{x\vec{a}+z\vec{c}}{x+z}\right)$

$x+y+z=0$

$y=-(x+z)$

(III) are of $\Delta ABC = 0$

Ques: $\begin{vmatrix} 2 & 5 & -4 \\ 1 & 4 & -3 \\ 4 & 7 & -6 \end{vmatrix}$

$= 2(-24+21) - 5(-6+12) - 4(7-16)$

$= -6-30+4(9)$

$= 36-36 = 0$ (collinear)

Ques: $\begin{vmatrix} 3 & -4 & 3 \\ -4 & 5 & -6 \\ 4 & -7 & 6 \end{vmatrix}$

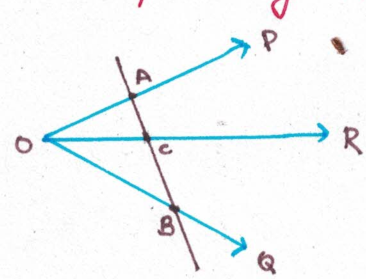
$= 3(30-42) + 4(-24+24) + 3(28-20)$

$= 3(-12) + 3(8)$

$= -36+24 \neq 0$ (Non-collinear)

Ques: $\vec{OP}$ and $\vec{OQ}$ have resultant $\vec{OR}$ and any transverse vector cuts their line of action at A, B, and C respectively. Prove that

$\dfrac{OP}{OA} + \dfrac{OQ}{OB} = \dfrac{OR}{OC}$



Sol: $\vec{OP} + \vec{OQ} = \vec{OR}$

$\dfrac{\vec{OP}}{|\vec{OA}|}\vec{a} + \dfrac{\vec{OQ}}{|\vec{OB}|}\vec{b} = \dfrac{\vec{OR}}{|\vec{OC}|}\vec{c}$

$\Rightarrow \dfrac{\overrightarrow{OP}}{|\overrightarrow{OA}|}\vec{a} + \dfrac{\overrightarrow{OQ}}{|\overrightarrow{OB}|}\vec{b} - \dfrac{\overrightarrow{OR}}{|\overrightarrow{OC}|}\vec{c} = 0$

Since, A, B, C are collinear

so, $\dfrac{|\overrightarrow{OP}|}{|\overrightarrow{OA}|} + \dfrac{|\overrightarrow{OQ}|}{|\overrightarrow{OB}|} = \dfrac{|\overrightarrow{OR}|}{|\overrightarrow{OC}|}$

SCALAR PRODUCT / DOT PRODUCT

Let there be 2 non-zero vectors A and B, then the product of these 2 vectors which result a scalar quantity is called dot product and is denoted as

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$ $\quad \theta \in [0, \pi]$

In general, the dot product is used to obtain the angle b/w 2 vectors.

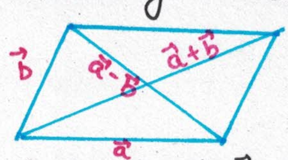
eg. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\qquad \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$
$$= \sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}\cos\theta$$

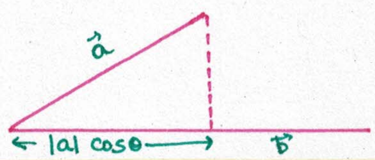
POINTS TO REMEMBER

- If $\vec{a}$ & $\vec{b}$ are 2 non-zero vectors and $\vec{a}\cdot\vec{b} > 0$ then angle b/w them is acute. If $\vec{a}\cdot\vec{b} < 0$, then angle is obtuse and if $\vec{a}\cdot\vec{b} = 0$, then $\theta = 90°$
- $(\vec{a} \pm \vec{b})^2 = |\vec{a}|^2 + |\vec{b}|^2 \pm 2\vec{a}\cdot\vec{b}$
- $\vec{a}\cdot\vec{a} = |\vec{a}|^2 = (\vec{a})^2$
- The sum of square of diagonal of a parallelogram equal to 2 times the sum of square its sides

  $(\vec{a}+\vec{b})^2 + (\vec{a}-\vec{b})^2 = 2[(\vec{a})^2 + (\vec{b})^2]$



- If magnitude of $\vec{a}+\vec{b}$ is equal to magnitude of $\vec{a}-\vec{b}$ then $\vec{a}$ is perpendicular to $\vec{b}$
- Dot product is also used to find the projection of vector on another



Projection of $\vec{a}$ on $\vec{b}$

$= a\cos\theta\ b$

$= \dfrac{\vec{a}\,\vec{b}}{|\vec{b}|} = \vec{a}\cdot\hat{b}$

Ques: $|\vec{a}| = 11$, $|\vec{b}| = 23$, $|\vec{a}-\vec{b}| = 30$, $|\vec{a}+\vec{b}| = ?$

Sol: $(30)^2 = 121 + 529 - (22)(33)\cos\theta$

$900 - 650 = -(22)(33)\cos\theta$

$250 = -(22)(33)\cos\theta$

$\vec{a}+\vec{b} = \sqrt{650 + (22)(33)\cos\theta}$

$= \sqrt{650 - 250} = \sqrt{400} = 20$

Ques: If $\vec{a}+\vec{b}+\vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 1$, $|\vec{c}| = 4$ then find $\Sigma\,\vec{a}\cdot\vec{b}$

Sol: Let $\vec{a} = 3\hat{i}$, $\vec{b} = \hat{i}$, $\vec{c} = -4\hat{i}$

$\Sigma\,\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$

$= 3\times1\times1 + 1\times4(-1) + 4\times3(-1)$

$= -13$

OR $\vec{a}+\vec{b}+\vec{c} = 0$

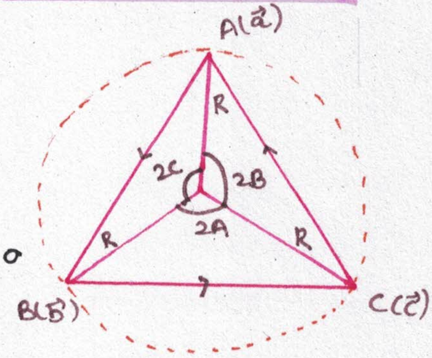
$a^2+b^2+c^2+\Sigma\,2(\vec{a}\cdot\vec{b}) = 0$

$\Sigma\,\vec{a}\cdot\vec{b} = -13$

NOTE Any vector $A(a_1, a_2, a_3)$ can be expressed as a scalar dot product in the form as-

$$\vec{a} = (\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$$

Ques: In a $\triangle ABC$, prove that $\Sigma \cos 2A > -3/2$



Sol: $|\vec{a}+\vec{b}+\vec{c}| \geq 0$

$\Rightarrow a^2+b^2+c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) \geq 0$

$\Rightarrow R^2+R^2+R^2 + 2(R^2\cos 2A + R^2\cos 2B + R^2\cos 2C) \geq 0$

$\Rightarrow 3R^2 + 2R^2(\Sigma\cos 2A) \geq 0$

$\Rightarrow \Sigma\cos 2A \geq -\frac{3}{2}$

Ques: $x, y \in R$ such that $2\sin x\sin y + 3\cos y + 6\cos x\sin y = 7$. Find $\tan^2 x + 2\tan^2 y$

Sol: $\vec{A} = 2\hat{i}+3\hat{j}+6\hat{k}$. $\vec{B} = \sin x\sin y\,\hat{i} + \cos y\,\hat{j} + \cos x\sin y\,\hat{k}$

$\vec{A}\cdot\vec{B} = (7)\sqrt{\sin^2 x\sin^2 y + \cos^2 y + \cos^2 x\sin^2 y} = 7$

$7\sqrt{1}\cos\theta = 7$

$\cos\theta = 1$, $\theta = 0°$

$\vec{A} = \lambda \vec{B}$

$2\hat{i} + 3\hat{j} + 6\hat{k} = \lambda(\sin x \sin y\, \hat{i} + \cos y\, \hat{j} + \cos x \sin y\, \hat{k})$

$$\frac{2}{\sin x \sin y} = \frac{6}{\cos x \sin y}$$

$\frac{1}{3} = \tan x$

$\tan^2 x = \frac{1}{9}$

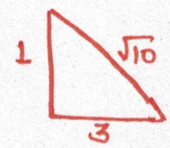
$\cos x = \frac{3}{\sqrt{10}}$

$\frac{\cos x \sin y}{\cos y} = \frac{6}{3}$

$\cos x \tan y = 2$

$\frac{3}{\sqrt{10}} \tan y = 2$

$2\tan^2 y = \frac{4(10)}{9} \times 2$

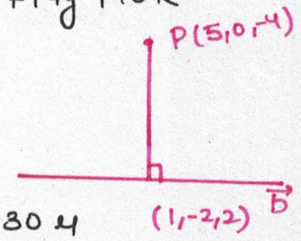


$\Rightarrow \quad \frac{1}{9} + \frac{80}{9} = \frac{81}{9} = 9$

Ques: A line passing through $\langle 1,-2,-1\rangle$ and parallel to $\vec{b}\langle 1,-2,2\rangle$. Find the distance of the pt. $\langle 5,0,-4\rangle$ from the line and also find the reflection of pt. P with respect to line.

Sol: $\vec{r} = \langle 1,-2,-1\rangle + \lambda\langle 1,-2,2\rangle$

$$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 5 & 0 & -4 \end{vmatrix} = \hat{i}(8) - \hat{j}(-4-10) + \hat{k}(10) = -8\hat{i} + 14\hat{j} + 10\hat{k}$$



$\vec{AP} = \langle 5,0,-4\rangle + \mu\langle 8,14,10\rangle$

$1+\lambda = 5+8\mu$

$\lambda = 4+8\mu$

$\lambda = 4 - \frac{8}{3} = \frac{16}{3}$

$-2-2\lambda = 14\mu$

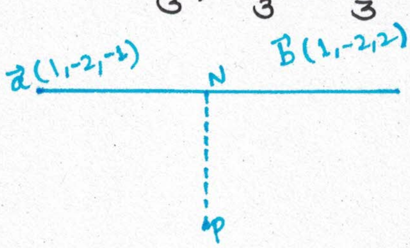
$-2-8-16\mu = 14\mu$

$-10 = 30\mu$

$\mu = -\frac{1}{3}$

① Image

$A = \frac{2}{3}, \frac{-4}{3}, \frac{-5}{3}$



$L: \vec{r} = (1,-2,-1) + \lambda(1,-2,2)$

$\vec{N} = (1+\lambda, -2-2\lambda, -1+2\lambda)$

$\vec{NP} = PV(\vec{P}) - PV(\vec{N})$

$\vec{NP} = (4-\lambda, 2+2\lambda, -3-2\lambda)$

$\vec{NP} \cdot \vec{b} = 0$

$4-\lambda-4-4\lambda-6-4\lambda = 0$

$9\lambda = -6$

$\lambda = -2/3$

$\vec{N} = \left(\frac{1}{3}, \frac{-2}{3}, -\frac{7}{3}\right)$

$|\vec{NP}| = \sqrt{\left(\frac{14}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{5}{3}\right)^2}$

$= \frac{1}{3}\sqrt{196+4+25}$

$= \frac{1}{3}\sqrt{225} = 5$

N is the mid pt. of PP'

$\frac{P'+P}{2} = N \Rightarrow P' = 2N - P$

$P' = \left(\frac{2}{3}, \frac{-4}{3}, \frac{-14}{3}\right) - (5, 0, -4) = \left(\frac{-13}{3}, \frac{-4}{3}, \frac{-2}{3}\right)$

Ques: If $a > 2$ and A, B and C are variable such that $\sqrt{a^2-4}\tan A + a\tan B + \sqrt{a^2+4}\tan C = 6a$. Find the min value of $\tan^2 A + \tan^2 B + \tan^2 C = Z$.

Sol: $\vec{A} = \sqrt{a^2-4}\,\hat{i} + a\hat{j} + \sqrt{a^2+4}\,\hat{k}$

$\vec{B} = \tan A\,\hat{i} + \tan B\,\hat{j} + \tan C\,\hat{k}$

$|\vec{A}| = \sqrt{a^2-4+a^2+a^2+4} = a\sqrt{3}$

$|\vec{B}| = \sqrt{Z}$

$a\sqrt{3}(\sqrt{Z})\cos\theta = 6a$

$\Rightarrow \sqrt{Z}\cos\theta = \frac{\sqrt{3}\sqrt{3}\,2}{\sqrt{3}}$

$\Rightarrow \sqrt{Z}\cos\theta = 2\sqrt{3}$

$\Rightarrow Z\cos^2\theta = 12$

$\Rightarrow Z_{min} = 12$

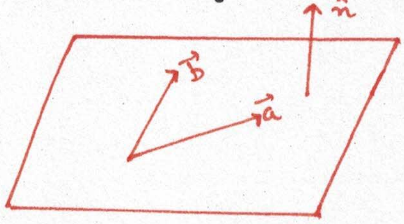
CROSS PRODUCT/VECTOR PRODUCT

The multiplication of 2 non-zero vectors in such a way that the resultant is also a vector called vector multiplication or cross multiplication.

$$\boxed{\vec{a}\times\vec{b} = ab\sin\theta\,\hat{n}}$$

Here, θ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is direction of perpendicular to plane of $\vec{a}$ & $\vec{b}$.

The direction of $\hat{n}$ will be given by Right hand thumb rule.



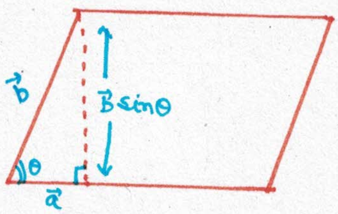
$\vec{a}\times\vec{b} = ab\sin\theta\,\hat{n}$

$\hat{n} = \frac{\vec{a}\times\vec{b}}{ab\sin\theta}$

$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

The cross product of 2 non-zero vectors always gives us the vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$ which is also perpendicular to both the given vectors.

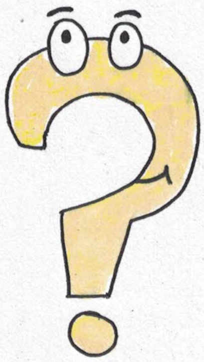
Geometrically, the cross prdt resembles the area vector of a parallelogram.



Area of parallelogram = Base X Height

$= |\vec{a}|\,|\vec{b}|\sin\theta$

$= |\vec{a}\times\vec{b}|$



Ques: $\vec{U} = \langle 2,-1,1\rangle$, $\vec{V} = \langle 1,1,1\rangle$ & W is a unit vector then $[U\ V\ W]$ will attain max. value?

Sol: $[U\ V\ W] = (\vec{U}\times\vec{V})\cdot\vec{W}$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= i(-1-1) - j(2-1) + \hat{k}(2+1)$

$= 2\hat{i} - \hat{j} + 3\hat{k}$

$= \sqrt{14}\cdot 1\cdot \cos\phi$

$[U\ V\ W] = \sqrt{14}\cdot 1\cdot\cos\phi$

$[U\ V\ W] = \sqrt{14}$

Ques: Prove $[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

Sol: $[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{c}\ \vec{a}]$

$= 2[\vec{a}\ \vec{b}\ \vec{c}]$

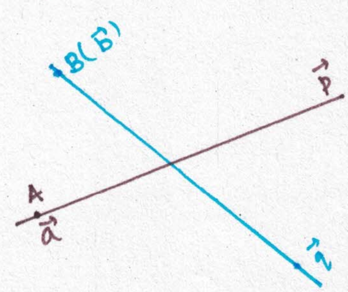
Volume of parallelopiped formed by diagonals of a given parallelopipe is twice the volume of given parallelopipe.

Ques: $\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + \mu\vec{q}$ are intersecting, then find the ratio of $\dfrac{[a\ p\ q]}{[b\ p\ q]}$

Sol: $[\vec{AB}\quad \vec{P}\quad \vec{q}] = 0$

$[\vec{b}-\vec{a}\quad \vec{p}\quad \vec{q}] = 0$

$[\vec{b}\ \vec{p}\ \vec{q}] - [\vec{a}\ \vec{p}\ \vec{q}] = 0$



$$\frac{[\vec{b}\ \vec{p}\ \vec{q}]}{[\vec{a}\ \vec{p}\ \vec{q}]} = 1$$

Ques: Prove that $[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\ \cdot\ \vec{c}\times\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

Sol: $(\vec{a}\times\vec{b})\ [(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$

$\Rightarrow (\vec{a}\times\vec{b})\ [u\times(\vec{c}\times\vec{a})]$

$\Rightarrow (\vec{a}\times\vec{b})\ [(\vec{u}\cdot\vec{a})\vec{c} - (\vec{u}\cdot\vec{c})\vec{a}]$

$\Rightarrow ((\vec{b}\times\vec{c})\cdot\vec{a})\vec{c} - ((\vec{b}\times\vec{c})\cdot\vec{c})\vec{a}$

$\Rightarrow (\vec{a}\times\vec{b})\cdot[\vec{a}\ \vec{b}\ \vec{c}]\vec{c}$

$\Rightarrow [a\ b\ c]\ \{(\vec{a}\times\vec{b})\vec{c}\}$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]^2$

Ques: $\vec{a} = 1,4,3 \qquad \vec{b} = 3,-2,6 \qquad \vec{c} = 2,-1,4.$ Find the $\vec{d}$ such that $\vec{d}$ is perpendicular to $\vec{a}$ & $\vec{b}$ & $\vec{c}\cdot\vec{d} = 15$

Sol: $\vec{d} = \lambda(\vec{a}\times\vec{b})$

$$\begin{vmatrix} 1 & 4 & 3 \\ 3 & -2 & 6 \end{vmatrix} = \hat{i}(24+6) - \hat{j}(6-9) + \hat{k}(-2-12)$$

$= 30\hat{i} + 3\hat{j} - 14\hat{k}$

$60\lambda\hat{i} + 3\lambda\hat{j} - 14\lambda\hat{k} = 15$

Ques: Find $\vec{r}$ such that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a} = 0$ where $\vec{a} = 2,0,1$, $\vec{b} = 1,1,1 \qquad \vec{c} = 4,-3,7$

Sol: $\vec{r}\times\vec{b} = \vec{c}\ \vec{b}$

$\vec{b}(\vec{r}-\vec{c}) = 0 \quad \Rightarrow \quad \vec{r}-\vec{c} = \lambda\vec{b}$

$\vec{r} = \vec{c} + \lambda\vec{b} = \langle 4,-3,7\rangle + \lambda\langle 1,1,1\rangle$

$\vec{r} = \langle 4+\lambda,\ -3+\lambda,\ 7+\lambda\rangle$

$\vec{r} = \langle 3+2\lambda + 0 + 7 + \lambda\rangle = 0$

$\lambda = -5$

$\vec{r} = \langle -1,\ -8,\ 2\rangle$

Ques: Find a unit vector $\vec{a}$ such that $\vec{a}\times(\hat{i}+2\hat{j}+\hat{k}) = \hat{i}-\hat{k}$

Sol: $\vec{a}\times(\hat{i}+2\hat{j}+\hat{k}) = \hat{j}(\hat{i}+2\hat{j}+\hat{k})$

$(\vec{a}-\hat{j})\times(\hat{i}+2\hat{j}+\hat{k}) = 0$

$\vec{a}-\hat{j} = \lambda(\hat{i}+2\hat{j}+\hat{k})$

$\vec{a} = \lambda\hat{i} + \hat{j}(1+2\lambda) + \lambda\hat{k}$

$\lambda^2 + (1+2\lambda)^2 + \lambda^2 = 1$

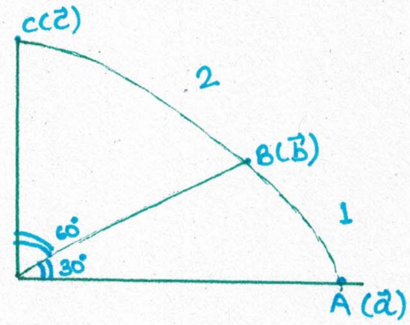
$6\lambda^2 + 4\lambda = 0$

$\lambda(6\lambda+4) = 0$

$\lambda = 0 \quad , \quad \lambda = -2/3$

Ques: An arc AC of the circle with the centre (0,0) and radius 1, Subtend 90° at origin. B divide AC in 1:2. Then find $\vec{c}$ in terms of $\vec{a}$ & $\vec{b}$, where they are position vectors.

Sol:



$\vec{c} = x\vec{a} + y\vec{b}$

$\vec{c}\cdot\vec{a} = x\vec{a}\cdot\vec{a} + y\vec{b}\cdot\vec{a}$

$0 = x + \frac{\sqrt{3}}{2}y \Rightarrow x = -\frac{\sqrt{3}}{2}y$

$\vec{c}\cdot\vec{c} = x\vec{a}\cdot\vec{c} + y\vec{b}\cdot\vec{c}$

$1 = y/2 \Rightarrow y = 2$

$x = -\sqrt{3}$

$\vec{c} = -\sqrt{3}\vec{a} + 2\vec{b}$

Ques: If A, B, C are 3 non-zero coplanar vectors, then prove that -

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\vec{c} \end{vmatrix} = 0$$

Sol: $\vec{c} = x\vec{a} + y\vec{b} = \begin{vmatrix} \vec{a} & \vec{b} & x\vec{a}+y\vec{b} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot(x\vec{a}+y\vec{b}) \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\,(x\vec{a}+y\vec{b}) \end{vmatrix}$

$C_3 = C_3 - xC_1 - yC_2$

$$\begin{vmatrix} \vec{a} & \vec{b} & 0 \\ \vec{a}\vec{a} & \vec{a}\cdot\vec{b} & 0 \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & 0 \end{vmatrix} = 0$$

Ques: $\vec{r} = (1,-2,3) + \lambda(-1,1,-2)$ $\quad \vec{r} = (1,-1,-1) + \mu(1,2,-2)$. Find the shortest distance between these lines.

Sol: $\vec{AB} = \vec{b} - \vec{a} = \hat{j} - 4\hat{k}$ $\qquad \vec{n} = \vec{p}\times\vec{q}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= i(-2+4) - j(2+2) + \hat{k}(-2-1)$$

$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

Shortest distance $= \left| \dfrac{(\vec{AB})(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$

$$= \left| \frac{(\hat{j} - 4\hat{k})(2\hat{i} - 4\hat{j} - 3\hat{k})}{\sqrt{4+16+9}} \right|$$

$$= \frac{|-4\hat{j} + 12\hat{k}|}{\sqrt{29}}$$

$$= \frac{8}{\sqrt{29}}$$